22/12/25

Paper / Subject Code: N10411 / Applied Mathematics-I

FE|sem-I / All Branches / NEP-2020 / NOV-DEC 2025

Marks-60

Q.P. code : 99303

(02-Hours)

Question 1 is compulsory.
Attempt any 3 from questions 2 to 6.
A Scientific Calculator is allowed to use.

Q-1 Attempt any **Five** questions. **(Compulsory Problem)** 15

a) If $u=\frac{\sqrt{x}+\sqrt{y}}{x+y}$ find $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$. 03

b) If $z=\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}$ and $\bar{z}$ is the conjugate of z, then prove that $(z)^{10}+(\bar{z})^{10}=0$. 03

c) Find n^{th} derivative of $y=sin2x\ cos6x$. 03

d) Express the matrix $A=\begin{bmatrix}2+3i & 2 & 3i\\ -2i & 0 & 1+2i\\ 4 & 2+5i & -i\end{bmatrix}$ as the sum of a Hermitian and a skew-Hermitian matrix. 03

e) Prove that $\log\left(\frac{x+iy}{x-iy}\right)=2itan^{-1}\left(\frac{y}{x}\right)$. 03

Solve $10x+y+z=12,\ x+10y+z=12\ ,x+y+10z=12$ using the Gauss-Jacobi's method by taking two iterations. 03

Q2-a) If $z=f(x,y),\ x=rcos\theta, y=rsin\theta$ then prove that $\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2=\left(\frac{\partial z}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$. 04

Q2-b) Using the Newton-Raphson method find the root of the equation $2x^3-3x+4=0$ lying between -2 and -1, correct to four places of decimals. 05

Q2-c) If $\tan(\alpha+i\beta)=e^{i\theta}$, prove that $\alpha=\frac{n\pi}{2}+\frac{\pi}{4}$ and $\beta=\frac{1}{2}log\left[tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right]$. 06

Q3-a) Find the rank of the matrix by reducing it to normal form $A=\begin{bmatrix}1 & -1 & -2 & -3\\ 4 & 1 & 0 & 2\\ 0 & 3 & 1 & 4\\ 0 & 1 & 0 & 2\end{bmatrix}$. 04

Q3-b) For $u=sin^{-1}(x^2+y^2)^{1/5}$ deduce $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{2}{25}tanu(2tan^2u-3)$. 05

Q3-c) If $sin^4\theta cos^3\theta=a_1cos\theta+a_3cos3\theta+a_5cos5\theta+a_7cos7\theta$, Then prove that $a_1+9a_3+25a_5+49a_7=0$. 06

Q4-a) Test the consistency and hence solve the following system of linear equations, $-2y+3t=2,\ 2x+y+z+t=-4, 4x-3y+z+7t=8$. 04

Q4-b) Show that $\log\left[\sin(x+iy)\right]=\frac{1}{2}log\left(\frac{cosh2y-cos2x}{2}\right)+itan^{-1}(cotx\ tanhy)$. 05

Q4-c) If $u=\log\left(x^3+y^3+z^3-3xyz\right)$, prove that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2u=-\frac{9}{(x+y+z)^2}$. 06

Q-5a) For the matrix $A=[a_{ijj}]_{3\times3}$, where $a_{ij}=i+j$, find the rank. 04

Q5-b) Find all the values of $\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{3/4}$ and show that their continued product is 1. 05

Q5-c) Discuss the maxima and minima of $x^3y^2(1-x-y)$. 06

Q6-a) Solve the following equations by Gauss-Seidel method by taking two iterations. $28x+4y-z=32,\ \ 2x+17y+4z=35,\ \ x+3y+10z=24$. 04

Q6-b) If $y=e^{msin^{-1}x}$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+m^2)y_n=0$ 05

Q6-c) Investigate for what values of λ and μ the system of equations $x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$ will have **(i) No Solution, ii) Unique Solution iii) Many Solutions.** 06

....