Paper / Subject Code: N2093111 / Applied Mathematics forCivil Engineering-I

SE/ CIVIL / SEM-III / NEP-2020 / DEC-23 17/12/23

Q.P.code:- 00530

Duration: 2 hours **Marks: 60**

Note: 1. Question **No.1** is compulsory

2. Attempt **any three** questions from the remaining.

3. Figures to the right indicate full marks

Q.1. Attempt **any 05** questions **[3 marks each]**

a) Find $L\left\{\frac{\cos 2t \sin t}{e^t}\right\}$

b) Find $L^{-1}\left\{\frac{3s-7}{s^2-6s+8}\right\}$

c) Find the Fourier series for $f(x) = \begin{cases} 1, & 0 < x < \pi \\ 2, & \pi < x < 2\pi \end{cases}$

d) Determine constants a, b, c and d if $f(z) = (x^2 + 2axy + by^2) + i(cx^2 + 2dxy + y^2)$ is analytic.

e) If $A = \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}$, then find the eigen values of $6A^{-1} + A^3 + 2I$.

f) Solve $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial t} = 0$ by Bender-Schmidt method, subject to the conditions $u(0,t) = 0, u(4,t) = 0,$ $u(x,0) = x(4-x)$. Taking $h = 1$ find the value of u upto $t = 3$.

Q.2. a) Find the Laplace Transform of $te^{3t}\sin 4t$. [4]

b) Construct an analytic function whose real part is $e^x \cos y$. [5]

c) Using Convolution theorem, find the Inverse Laplace Transform of $\frac{1}{(s-2)(s+2)^2}$ [6]

Q.3. a) Find $L\left\{\frac{e^{-t}\sin t}{t}\right\}$ [4]

b) Find Inverse Laplace Transform $\frac{s+29}{(s+4)(s^2+9)}$ [5]

c) Find the Fourier expansion of $f(x) = 2x - x^2, 0 \le x \le 3$ whose period is 3. [6]

Q.4. a) Determine the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the boundary conditions $u(0,t) = 0, u(l,t) = 0$ and $u(x,0) = x, 0 < x < l, l$ being the length of the rod. [4]

b) Evaluate $\int_0^\infty \frac{\cos at - \cos bt}{t} dt$ [5]

c) Find eigen values and eigen vectors of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ [6]

Q.5.

a) Show that the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ is diagonalisable and hence find the diagonal form. [4]

b) Expand $f(x) = \begin{cases} \pi x, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$ with period 2into a Fourier Series. [5]

c) If the imaginary part of the analytic function $w = f(z)$ is $v = x^2 - y^2 + \frac{x}{x^2+y^2}$ then show that its real part is $u = -2xy + \frac{y}{x^2+y^2} + c$. [6]

Q.6.

a) Obtain half-range sine series of $f(x) = x(\pi - x)$ in $(0, \pi)$. [4]

b) Use Cayley-Hamilton theorem to find $2A^4 - 5A^3 - 7A + 6I$, where $A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$ [5]

c) Using Crank-Nicholson Simplified formula solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$, subject to the conditions $u(0,t) = 0$, $u(4,t) = 0$ and $u(x,0) = \frac{x}{3}(16 - x^2)$. Find u_{ij} for $i = 0, 1, 2, 3, 4$ and $j = 0, 1, 2$. [6]

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