Max Marks: 60

(Time: 2 hours)

Note: 1. Question No. 1 is Compulsory.
2. Attempt any 3 Questions from the remaining questions.
3. Scientific Calculator is allowed to use

Que. 1   Attempt **any Five** questions.

a. Solve $(x - 2e^y)dy + (y + x\ sinx)dx = 0$ 3

b. Using Euler's method find approximate value of y when $\frac{dy}{dx} = x^2 + y^2$;   3
and $y(0) = 1$ at $x = 0.6$ take $h = 0.2$.

c. Evaluate: $\int_0^2 \int_0^z \int_0^{yz} xyz\ dxdydz$.   3

d. Evaluate Gamma function $\int_0^\infty x^2 e^{-x^8}\ dx$.   3

e. Evaluate: $\int_0^1 \int_0^x e^{x+y}\ dydx$.   3

f. Solve $(\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y) = 0$ 3

Que. 2   a. Solve $\frac{dy}{dx} = x + y$ with $x_0 = 0, y_0 = 1$ by Euler's modified method for   4
$x = 0.05$ correct to three places of decimals. (take h = 0.05)

b. Show that $\int_0^{2a} x^2\sqrt{2ax - x^2}\ dx = \frac{5}{8}a^4\pi$.   5

c. Use method of variation of parameters to solve the differential equation   6
$(D^2 - 2D + 2)y = e^x\ tanx$.

Que. 3   a. Evaluate $\int_0^\pi \int_0^{a\ sin\theta} r\ drd\theta$.   4

b. Solve the differential equation $(x^2 + y^2 + 1)dx - 2xydy = 0$ 5

c. Solve $\frac{dy}{dx} = \frac{1}{x+y}, x_0 = 0, y_0 = 1$ for the interval (0,1) choosing h=0.5 by   6
Runge–Kutta method of 4th Order (up to 3 decimal places).

Que. 4   a. Solve $(D^3 + D)y = x^2$ 4

b. Find the mass of the lamina bounded by the curves $ay^2 = x^3$ and the   5
line $by = x$ if the density at a point varies as the distance of the point from the
x-axis.

c. Solve $\frac{dy}{dx} = 2y(1 - 2xy)$ 6

Que. 5   a. Prove that $\int_0^\infty \frac{log(1+ax^2)}{x^2}\ dx = \pi\sqrt{a}, (a > 0)$.   4

b. Find the area of the cardioid $r = a(1 + cos\theta)$ by double integration.   5

c. Evaluate by changing into polar coordinates $\int_0^a \int_y^a \frac{x^2}{\sqrt{x^2+y^2}}\ dxdy$.   6

Que. 6   a. Change the order of integration $\int_0^1 \int_0^{\sqrt{1-y^2}} f(x,y)dxdy$.   4

b. Solve the differential equation $(D^2 - 3D + 2)y = e^{2x}x^2$ 5

c. Evaluate $\int_0^1 \frac{x^2}{1+x^3}dx$ by using Trapezoidal rule and Simpson's $(1/3)^{rd}$ 6
rule. (choose h = 0.25)