Marks : 60

Time: 2 Hour

Note: 1. Question No. 1 is Compulsory.
2. Attempt any 3 Questions from the remaining questions.
3. Scientific Calculator is allowed to use

Que. 1    Attempt **any Five** questions of the following.

a. Find Laplace transform of $e^{-3t} \cosh 5t \sin 4t$    3

b. Find inverse Laplace transform of $\frac{s+2}{s^2-2s+17}$    3

c. Obtain half range sine series for $f(x) = x$ in $(0, \pi)$    3

d. Show that $v = e^x \sin x$ is a harmonic function.    3

e. If $\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ then find eigen values of $4A^{-1} + 3A + 2I$    3

f. An elastic string is stretched between two points in it's equilibrium position a point, it displayed through a distance transversely &then released from this position, State it's equation.    3

Que. 2   a. Find analytic function whose real part is $\frac{1}{2}\log(x^2 + y^2)$    4

b. Find half range cosine series for $f(x) = x$ in $(0, 2)$    5

c. By Laplace transform, prove that $\int_0^\infty e^{-t} \frac{\sin^2 t}{t} dt = \frac{1}{4}\log 5$    6

Que. 3   a. Find inverse Laplace transform of $\log\left(1 + \frac{1}{s^2}\right)$    4

b. Determine Constants a, b, c and d, if
$f(z) = (x^2 + 2axy + by^2) + i(cx^2 + 2dxy + y^2)$
is an analytic function.    5

c. Find the values of u(x, t) satisfying differential equation $\frac{\partial^2 u}{\partial x^2} - \frac{1}{4}\frac{\partial u}{\partial t} = 0$    6
with boundary conditions $u(0, t)=0$, $u(8, t)=0$ $u(x, 0) = 4x - \frac{x^2}{2}$ at the
point x=i, where i=0, 1, 2, 3, 4, 5, 6,7,8 and $t = \frac{j}{8}$ where j=0, 1, 2, 3, 4, 5, using Bender- Smith relation

Que. 4    a.    Find Fourier series of $f(x) = x|x|$ in $(-1, 1)$      4

         b.    Using Crank- Nicholson simplified formula, solve      5

$\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ with boundary conditions $u(0, t) = 0$, $u(4, t) = 0$

$u(x, 0) = \frac{x}{3}(16 - x^2)$ at the point x=i, where i=0, 1, 2, 3, 4, and

t = j where j = 0, 1

         c.    Find eigen values and eigen vectors of $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$      6

Que. 5    a.    Verify Cayley- Hamilton theorem for matric      4

$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

         b.    Find Laplace transform of $g(4t)$ where $g(t) = t\sqrt{1 + \sin t}$      5

         c.    Expand Fourier series for $f(x) = x^2$ in $(0, 2\pi)$      6

Que. 6    a    Find Laplace transform of $\int_0^t \int_0^t \int_0^t e^{-2u} \cos^2 u (du)^3$      4

         b.    Show that the matrix $A = \begin{bmatrix} 2 & -2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & -2 \end{bmatrix}$ is diagonalisable and find it's      5

diagonalising matrix

         c.    By using convolution theorem, find inverse Laplace transform of      6

$\frac{s^2}{(s^2+a^2)(s^2+b^2)}$ where a>0 and b>0 are constants.

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